Robust Motion Control of a Four Wheel Drive Skid-Steered Mobile Robot

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Abstract

In this paper robust motion control of a four wheel drive skid-steered mobile robot (4WD SSMR) is presented. We have developed a motion control system where a kinematic trajectory tracking controller based on the vector field orientation (VFO) strategy and a robust dynamic velocity controller based on the sliding mode control (SMC) technique and the computed torque method (CTM) are combined. Asymptotic stability for a class of reference trajectories is guaranteed by the VFO method while the stability of the velocity controller is based on the Lyapunov theory. In addition to the original VFO method a multi parameter orienting control is used. A 4WD SSMR is designed in a three dimensional (3D) realistic simulation environment to test the performance of the motion control system developed. Simulation results have shown the stability and robustness of the motion control system even under heavy perturbed conditions and the proposed multi parameter orienting control strategy has the advantage of smoother path tracking.

1. Introduction

Skid-steered mobile robots (SSMR) are well-known for their robust structure which is suitable for outdoor usage especially on rough terrains. SSMRs are differentially driven vehicles (DDVs) because they are rotated by differential speeds or torques on left and right side wheels. SSMRs do not have a mechanical steering system and lateral skid is necessary for the vehicle to change its heading direction. Because of this nature of skid-steered vehicles, wheel-ground interaction forces play an important role in the vehicle dynamics. The most important ones, lateral friction forces due to sliding of wheels on ground may be too high on a hard terrain such as an asphalt road. As a result it may become too difficult to control the yaw rate of the vehicle and require high torques generated by the vehicle actuators. Hence power loss due to high friction forces is inevitable.

Motion control or trajectory tracking for WMRs is often studied considering only kinematics omitting the dynamic properties of the vehicle assuming that wheels track commanded velocities perfectly and they do not slip while rolling. As a result it is assumed that the nonholonomic constraints of the vehicle are satisfied all the time during robot motion such that one can calculate wheel angular velocities for a desired robot motion. In fact parameter and non-parameter perturbations, external disturbances, modeling errors due to simplifying assumptions and unmodeled system dynamics have impacts on the robot motion directly or indirectly disturbing the slip-skid phenomena. Hence the wheels to body motion kinematic model does not hold all the time causing tracking errors. So in this study we consider a dynamic model based control strategy to robustly stabilize a class of reference trajectories which rejects aforementioned effects.

In 1996, Fierro and Lewis introduced a combined kinematic and torque control framework using backstepping technique to join robot kinematics into dynamics allowing one to apply control approaches from model dependent computed torque method (CTM) to robust sliding mode control (SMC) technique [1]. In 1999, Caracciolo et al. studied the dynamics of a 4WD SSMR offering a nonholonomic operational constraint on the instantaneous center of rotation (ICR) of the vehicle [2]. This additional nonholonomic operational constraint acts like an outer-loop controller term preventing excessive skidding of the vehicle by limiting the vehicle's lateral velocity with the yaw rate [3, 4]. In 2004, Kozlowski et al. redefined the kinematic and dynamic model of 4WD SSMR in [4] and [5] based on the model given in [2]. Later Michałek and Kozłowski introduced a novel VFO feedback control method for trajectory tracking of a DDV in [6]. Also Michałek et al. extended the VFO control method for the case of limited skid-slip phenomena in [7]. In 2008, a decoupling design approach using two new torque variables for controlling the linear velocity and yaw angle of a 4WD SSMR platform using SMC technique without the so-called operational constraint is introduced [8].

Here we propose a robust motion control system for the class of 4WD SSMR platforms based on the backstepping kinematics into dynamics framework and VFO trajectory tracking control method while controlling the vehicle's linear and angular velocities using a CTM plus SMC technique considering both system dynamics and kinematics.

A 4WD SSMR platform is being developed at the robotics laboratory in our department. The state of the art computer aided design (CAD) model of the mobile robot platform is shown in Fig. 1.

This paper is organised as follows. First the mathematical model of the vehicle is derived and next, a model based robust motion control system is designed. Then simulation results are given. Finally simulation results are analyzed and final remarks are made.

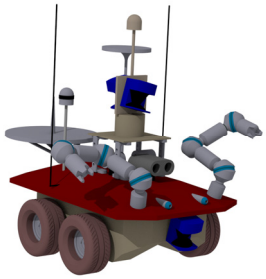

Fig. 1. 4WD SSMR platform

2. Mathematical Model

The vehicle model is derived based on the assumptions given in [2] and [4] except that the kinetic energy of wheels is not neglected here. We consider only planar motion. Each wheel-ground contact is a single point and normal forces acting on the wheel-ground contact points are constant depending on the mass of vehicle and gravity. We assume that wheels do not slip while rolling. Wheel-ground interaction forces are represented with a conventional coulomb friction model. Lateral friction is due to sliding and longitudinal friction is due to rolling of wheels on the ground. The vehicle body is represented with a point mass located at the center, near the front side of vehicle. The wheel torques are distributed equally on each side and servo drives track torque commands perfectly such that the electric drives' dynamics can be neglected.

2.1 Kinematics

Kinematics of mobile robot platform is illustrated in Fig. 2. The vehicle configuration vector in global coordinate frames is

$$q = \begin{bmatrix} X & Y & \theta \end{bmatrix}^T \in \mathbb{R}^3 \tag{1}$$

where X, Y and θ are the position and orientation of vehicle respectively. The transformation between the local velocities defined in the local coordinate frames attached on the vehicle's center of mass (COM) and the generalized velocities is

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ w \end{bmatrix} \tag{2}$$

COM is $d \in (0, b)$ distance away from the center of geometry (COG). ICR is located on the axis that intersects COG as shown in Fig. 2. In this case the nonholonomic operational constraint that limits lateral skid is defined as [2]

$$v_y - dw = 0 \tag{3}$$

$$\begin{bmatrix} -\sin\theta & \cos\theta & -d \end{bmatrix} \begin{bmatrix} \dot{X} & \dot{Y} & \dot{\theta} \end{bmatrix}^T = A(q)\dot{q} = 0 \tag{4}$$

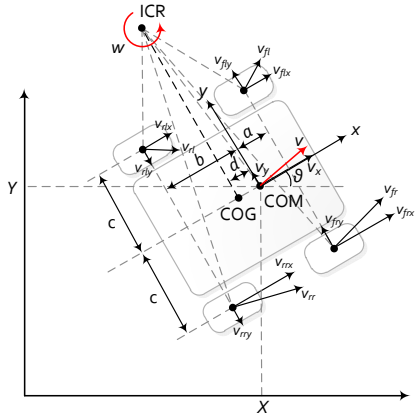

Fig. 2. 4WD SSMR kinematics

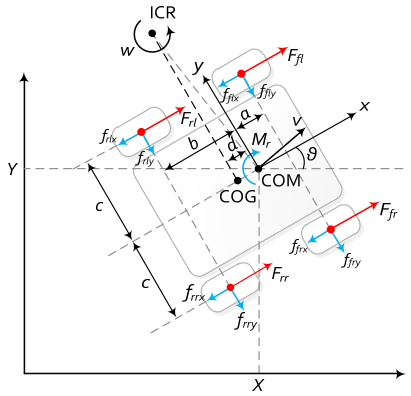

Fig. 3. 4WD SSMR dynamics

Then we can rewrite the Eq. (2) in the form below where $S(q) \in \mathbb{R}^{3x2}$ is a matrix and $\eta \in \mathbb{R}^2$ is called as the control input vector at kinematic level defined as

$$\dot{q} = S(q)\eta \tag{5}$$

$$S(q) = \begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}, \quad \eta = \begin{bmatrix} v_x & w \end{bmatrix}^T \tag{6}$$

and since the columns of $S(q)$ are always in the null space of $A(q)$ the following expression is satisfied [2, 4].

$$S^T A^T = 0 \tag{7}$$

2.2. Dynamics

Dynamics of mobile robot platform is illustrated in Fig. 3. First we introduce the left and right side forces then we describe the wheel dynamics

$$F_L/2 = F_{fl} = F_{rl}, \quad F_R/2 = F_{fr} = F_{rr} \tag{8}$$

$$I_w \dot{w}_w = \tau - DF \tag{9}$$

where I_w is the wheel inertia, $w_w = [w_{fl}\ w_{rl}\ w_{fr}\ w_{rr}]^T \in \mathbb{R}^4$ is the wheel angular speeds vector, $\tau = [\tau_{fl}\ \tau_{rl}\ \tau_{fr}\ \tau_{rr}]^T \in \mathbb{R}^4$ is the wheel torques vector, $F = [F_L\ F_R]^T \in \mathbb{R}^2$ is the force vector and $D \in \mathbb{R}^{4x2}$ is the force-torque conversion matrix defined as

$$D = \frac{r}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}^T \tag{10}$$

where r is the wheel radius. Next the equations of robot motion in global coordinates follow

$$m\ddot{X} = (F_L + F_R)\cos\theta - (f_x\cos\theta - f_y\sin\theta) \tag{11}$$

$$m\ddot{Y} = (F_L + F_R)\sin\theta - (f_x\sin\theta + f_y\cos\theta) \tag{12}$$

$$I\ddot{\theta} = c(-F_L + F_R) - M_r \tag{13}$$

where m is the vehicle's mass and I is the vehicle's inertia about z-axis, f_x and f_y are the rolling and sliding friction forces respectively and M_r is the resistive moment about z-axis. Rolling friction forces are too small when compared to sliding friction forces in ideal case. A realistic model for rolling friction in case of slip can be found, for example in [3]. We define the friction forces for only front-left wheel as follows

$$f_{flx} = \mu_x N_{fl} \operatorname{sgn}\left(v_{flx}\right),\ f_{fly} = \mu_y N_{fl} \operatorname{sgn}\left(v_{fly}\right) \quad (14)$$

where normal forces acting on the wheel-ground contact points due to gravity are calculated as

$$N_{fl} = N_{fr} = \frac{b}{a+b}\frac{mg}{2},\ N_{rl} = N_{rr} = \frac{a}{a+b}\frac{mg}{2} \quad (15)$$

and finally the resistive moment is calculated as follows.

$$\begin{aligned} M_r &= a\left[f_{fly} + f_{fry}\right] - b\left[f_{rly} + f_{rry}\right] \\ &+ c\left[f_{frx} + f_{rrx} - f_{flx} - f_{rlx}\right] \end{aligned} \quad (16)$$

The general form of the vehicle dynamics including the nonholonomic constraint using Euler-Lagrange principle and introducing an additional vector for representing disturbances is

$$M\left(q\right)\ddot{q} + R\left(\dot{q}\right) + F_d = B\left(q\right)F + A^T\lambda \quad (17)$$

where $M \in \mathbb{R}^{3x3}$ is the mass and inertia matrix, $R \in \mathbb{R}^3$ is the vector of resistive forces and torques, $F_d \in \mathbb{R}^3$ is the vector of disturbances, $B \in \mathbb{R}^{3x2}$ is the input matrix, F is called as the control input at dynamic level previously defined, A is the constraint vector as in Eq. (4), and λ is the vector of Lagrange multipliers.

$$M\left(q\right) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix},\ B\left(q\right) = \begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ -c & c \end{bmatrix},$$
$$R\left(\dot{q}\right) = \begin{bmatrix} f_x\cos\theta - f_y\sin\theta \\ f_x\sin\theta + f_y\cos\theta \\ M_r \end{bmatrix} \quad (18)$$

Taking the time derivative of Eq. (5) yields

$$\ddot{q} = S\left(q\right)\dot{\eta} + \dot{S}\left(q\right)\eta \quad (19)$$

and using the relationships Eq. (5), Eq. (7) and Eq. (19) one can convert the dynamic system in Eq. (17) to

$$\overline{M}\dot{\eta} + \overline{C}\eta + \overline{R} + \overline{F}_d = \overline{B}F \quad (20)$$

$$\overline{M} = S^T M S,\ \overline{C} = S^T M\dot{S},\ \overline{R} = S^T R,\ \overline{F}_d = S^T F_d,$$
$$\overline{B} = S^T B \quad (21)$$

3. Motion Control

Motion control of a vehicle is often regarded as trajectory tracking control. Here we regard it as the motion control system that has two sub-systems, that is, trajectory tracking controller at kinematic level and velocity controller at dynamic level. Such a control system is illustrated in Fig. 4.

3.1. Control Problem

The control objective is to asymptotically stabilize the trajectory tracking error at origin. Here we restrict the input trajectories to be persistently exciting and admissible. Let the reference trajectory be denoted by q_d then the trajectory tracking error is $e(t) \triangleq q_d(t)$ - q(t). Now define the admissible local velocity vector $\eta_d(t) \triangleq [v_d(t)\ w_d(t)]^T$ then the so-called persistently exciting reference trajectory is defined as

$$\dot{q}_d\left(t\right) \triangleq S\left(q_d\left(t\right)\right)\eta_d\left(t\right),\ v_d\left(t\right) > 0 \quad (22)$$

The trajectory tracking control problem is to find a smooth velocity control [1]

$$\eta_c\left(t\right) = f\left(e, \eta_d, K\right) \quad (23)$$

such that $e(t) \to 0$ as $t \to \infty$, where K is the controller design parameters vector.

3.2. Trajectory Tracking

A velocity control that achieves tracking for a DDV is given in [1]

$$\eta_c = \begin{bmatrix} v_{xd}\cos e_\theta + k_1 e_x \\ w_d + k_2 v_{xd} e_y + k_3 v_{xd}\sin e_\theta \end{bmatrix} \quad (24)$$

where k_1, k_2, $k_3 > 0$ are controller design parameters and e_x, e_y, e_θ are the components of the error vector in local coordinates.

Another approach, VFO is a motivating control technique to calculate such velocity commands for DDVs. The reader can refer to [6] (an application of VFO to a DDV) and [7] (an extension of VFO in the case of skid-slip phenomena) for more about VFO. In VFO strategy the trajectory tracking control problem is divided into two subtasks, convergence of position and orientation to their desired values. The vehicle is driven by the *pushing control* v_c with the *careful pushing* strategy while the *orienting control* w_c is responsible for matching the vehicle's heading vector with the position convergence vector. Such tasks are accomplished with the choice of proper *convergence vector field* which defines the instantaneous convergence direction and orientation for the vehicle.

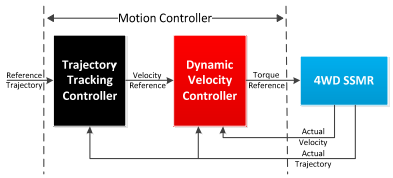

Fig. 4. Motion control system

For control purposes with VFO, we define a new position vector $q_c \triangleq [X_c\ Y_c]^T \in \mathbb{R}^2$, that is COG, then the new position tracking error becomes $e_c \triangleq q_{cd} - q_c$ where

$$\begin{bmatrix} X_c \\ Y_c \end{bmatrix} = \begin{bmatrix} X - d\cos\theta \\ Y - d\sin\theta \end{bmatrix},\ e_c = \begin{bmatrix} X_{cd} - X_c \\ Y_{cd} - Y_c \end{bmatrix} \quad (25)$$

Definition of the so-called *convergence vector field* is $h_c \triangleq [h_c^*\ h_o]^T \in \mathbb{R}^3$ where $h_c^* \in \mathbb{R}^2$ defines the convergence direction and orientation of q_c sub-state and h_o defines the convergence orientation of θ variable

$$h_c^* = k_c e_c + v_d S_1^*(\theta_d),\ h_o = k_1 e_a + \dot{\theta}_a \quad (26)$$

where $S_1^*(\theta_d) \in \mathbb{R}^2$ defines the instantaneous heading direction of the reference vehicle, e_a is the auxiliary angle error and θ_a is the so-called auxiliary angle

$$S_1^*(\theta_d) = [\cos\theta_d \quad \sin\theta_d]^T,\ e_a \triangleq \angle T^T h_c^* \quad (27)$$

$$T(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},\ \theta_a \triangleq \angle h_c^* = e_a + \theta \quad (28)$$

and we define the *pushing* and *orienting control* commands.

$$v_c \triangleq \left\| h_c^* \right\| \cos e_a,\ w_c \triangleq h_o \quad (29)$$

A geometrical VFO tracking scheme for k_c=k_1=1 is illustrated in Fig. 5. For system in Eq. (5) to track a reference trajectory in Eq. (22) it is sufficient to drive the vehicle with the velocity control $\eta_c(t) \triangleq [v_c(t)\ w_c(t)]^T$. This is guaranteed by the VFO method. The *orienting control* in Eq. (26) is similar to a PD plus control but the derivative parameter is fixed to one. Hence adjusting the controller performance is limited to a single proportional parameter. Here we propose a PID plus multi parameter *orienting control* defined as below.

$$w_c = k_1 e_a + k_2 \dot{e}_a + k_3 \int e_a dt + \dot{\theta} \quad (30)$$

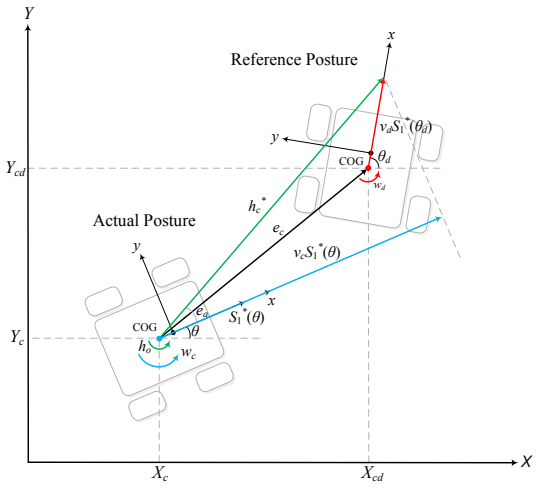

Fig. 5. VFO trajectory tracking scheme

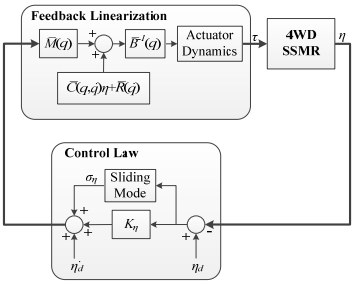

Fig. 6. Velocity control with SMC technique

3.3. Velocity Control

Now we need to construct a robust velocity control system that will track the desired velocities to complete the motion control system in Fig. 4. The block diagram of velocity control system is illustrated in Fig. 6. Applying the following nonlinear feedback to the system in Eq. (20) we have

$$u = \overline{M}^{-1}\left[\overline{B}F - \overline{C}\eta - \overline{R}\right] \quad (31)$$

$$\dot{\eta} = u - n(q, \eta, \dot{\eta}) \quad (32)$$

$$\tau = D\overline{B}^{-1}\left[\overline{M}u + \overline{C}\eta + \overline{R}\right] + I_w \dot{w}_w \quad (33)$$

where $n \triangleq [n_1\ n_2]^T \in \mathbb{R}^2$ is the vector of uncertainties and the control law is

$$u = \dot{\eta}_d + K_\eta e_\eta + \sigma_\eta \quad (34)$$

$$e_\eta \triangleq [e_v \quad e_w]^T \in \mathbb{R}^2,\ \sigma_\eta \triangleq [\sigma_v \quad \sigma_w]^T \in \mathbb{R}^2 \quad (35)$$

where $K_\eta \in \mathbb{R}^{2x2}$ is a design matrix, $e_\eta = \eta_d - \eta$ is the velocity error and σ_η is the sliding mode control law. Then one can write the velocity error dynamics

$$\dot{e}_\eta = -K_\eta e_\eta + (n - \sigma_\eta) \quad (36)$$

The Lyapunov candidate $V = (e_v^2 + e_w^2)/2$ is zero only for $e_\eta = 0$ and $V \geq 0$ for all $e_\eta \in \mathbb{R}^2$. Taking the time derivative of V and using the Eq. (36) yields

$$\dot{V} = -e_\eta^T K_\eta e_\eta + e_\eta^T [n - \sigma_\eta] \quad (37)$$

and we define the sliding surface $s_\eta \triangleq [s_v\ s_w]^T \in \mathbb{R}^2$ and the sliding mode control law as

$$s_\eta = \partial V / \partial e_\eta = [e_v \quad e_w]^T \quad (38)$$

$$\sigma_v = \rho_v \operatorname{sgn}(s_v),\ \sigma_w = \rho_w \operatorname{sgn}(s_w) \quad (39)$$

$$\rho_v \geq \|n_1\|,\ \rho_w \geq \|n_2\| \quad (40)$$

where ρ_v and ρ_w are the linear and angular velocity sliding mode control gains respectively chosen high enough to reject the disturbances and uncertainties in the system. It is clear that $\dot{V}$ is zero only for $e_\eta = 0$ and $\dot{V} \leq 0$ for all $e_\eta \in \mathbb{R}^2$ so the asymptotic convergence of e_η to zero is guaranteed.

4. Simulation

A 4WD SSMR is modeled in a 3D realistic environment using MD-ADAMS View, Tire and Road modules. The controller is implemented with MATLAB-Simulink interface. A circular reference trajectory is applied to test various control systems. The reference vehicle starts at location (2.5m, 0m) with yaw angle $\theta_d(0)=\pi/2$rad and moves along a circle with constant speeds. We assigned the vehicle parameters as m=150kg, I=30kgm^2, μ_x=0.003, μ_y=0.6, r=0.2m, a=0.2m, b=0.3m, c=0.45m, d=0.05m, the trajectory tracking controller parameters k_c=1, k_1=5, k_2=1, k_3=1 and the velocity controller gain K_η=[10 0; 0 10]. We assigned the sliding mode gains with a formula, $\rho_w=\rho_v\times c\times m/I$, ρ_v=100, ρ_w=225. In order to reduce the chattering effects we used low-pass filters, rate limiter and saturation blocks for F, σ_v and σ_w. Figure 7 shows the results for VFO and CTM while Fig. 8 shows the results for VFO and SMC based motion control system. In Fig. 8a trajectory tracking, in Fig. 8b the desired, commanded and actual velocities and in Fig. 8c the left and right side control forces are shown.

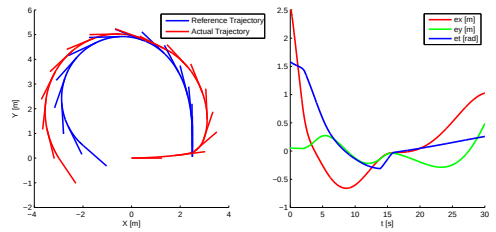

Fig. 7. VFO and CTM with 30% parameter perturbation

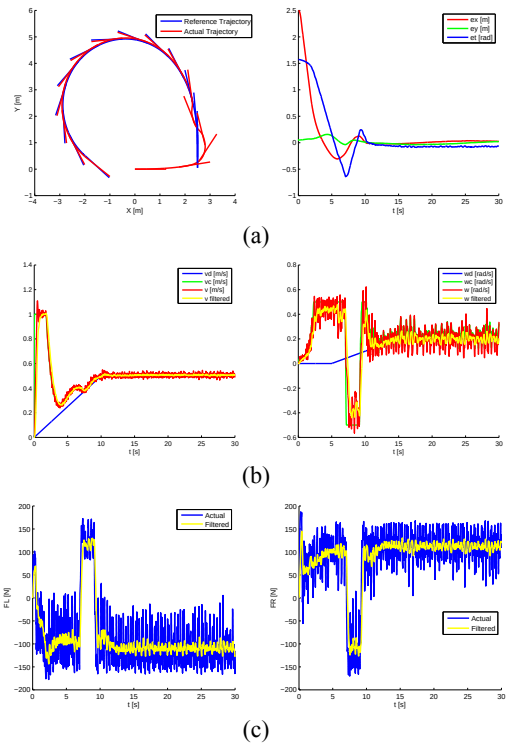

Fig. 8. VFO and SMC with 87.5% parameter perturbation (a) Trajectory tracking (b) Linear and angular velocities (c) Left and right side forces

5. Conclusions

In this paper we have developed a robust motion control system which consists of two sub-systems. In kinematic level VFO trajectory tracking method with orienting and pushing strategy is used. In dynamic level robustness is achieved by using CTM plus SMC technique which fully rejects disturbances, modeled and unmodeled system dynamics. A PID plus multi parameter orienting control is used instead of single parameter dependent one in the original VFO method. A 4WD SSMR is designed in CATIAv5, Siemens NX6 and simulated in MD-ADAMS multi body dynamics engine. Motion control systems are implemented in Simulink and interfaced with MD-ADAMS. Simulation results proved the stability and robustness of the motion control system for persistently exciting admissible reference trajectories even under heavy parameter uncertainty conditions. We also achieved a smoother path tracking by using the multi parameter orienting control.

Acknowledgement

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